Engineering Notes

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A Satellite Digital Controller

S.M. Seltzer*
NASA George C. Marshall Space Flight Center, Huntsville, Ala.

Introduction

THE problem addressed in this Note is the design of a digital controller for the attitude of a satellite spacecraft. A simplified model is used to portray the dynamics of the system. The controlled plant is considered to be a rigid body acting in a plane. Flexible body effects are neglected. It is assumed that the attitude ϕ and its time rate of change $\dot{\phi}$ are sensed by perfect onboard sensors. The controller is assumed to be an onboard digital computer. The objective is to present a design strategy and technique for selecting numerical values for both the control gains contained within a proposed control algorithm and the sample rate of the digital controller. It is proposed that the technique be applied during the preliminary design of a digital controller of a satellite when preliminary values are needed for the control gains and the sample rate. It is recognized that these values (and indeed the composition of the control algorithm itself) may have to be altered as the dynamic nature of the satellite's flexible appendages becomes known. Still it is postulated that this approach is superior to the "cut-and-try" approaches employed by many engineers and that the proposed simple position, integral, derivative (PID) feedback control algorithm may have engineering practicality over a multistate feedback controller resulting from "modern" optimal control theory. In effect, what is proposed in this paper is a sampled-data analog to the tuning of continuous-data PID controller gains which was so popular in the 1940's and 1950's and still persists within many industrial and chemical process journals.[1]

The technique is applied to a system whose equations of motion are stationary and linear and which may be cast in the complex z-transform domain. It is proposed that the advantage of this technique is its simplicity over extant methods. Techniques for selecting the digital PID control gains and the sample period T are described, first to ensure stability and then to provide a desired transient response within certain design limitations. An example of their application to the Space Telescope is provided.

System Model

The planar model of satellite spacecraft rotational dynamics which is used herein is shown in block diagram form (Fig. 1), where T_c represents the commanded torque. The onboard digital controller develops T_c from the input states ϕ and $\dot{\phi}$ and the commanded attitude ϕ_c; $G_{ho}(s)$ represents a zero-order hold in the computer. The PID control algorithm position, integral, and derivative feedback gains are K_p, K_i, and K_d.

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*Chief, Pointing Control Systems Branch, Systems Dynamics Laboratory. Associate Fellow AIAA.

Control Systems Design

The analytical tool used herein is based on the parameter plane technique for control-system analysis and synthesis of both continuous-data systems and sampled-data systems.[2,3] The method has been extended to portray the effects of varying the sampling period, and a set of recursive formulas is shown therein which are simpler than the Chebyshev functions used in Ref. 3.[4] The technique requires that the control system be described by a characteristic equation, which must be transformed into the z domain. By applying sampled signal flow graph techniques to the block diagram of Fig. 1, the third-order characteristic equation (C.E.) may be obtained in terms of the system parameters. Application of the referenced parameter plane technique yields a three-dimensional stability region in terms of modified gains a, b, and c, where $a \equiv K_d T/J_v$, $b \equiv K_p T^2/2J_v$, and $c \equiv K_i T^3/4J_v$. The stable region is bounded by the planes $c=0$ and $a=2$ and the surface $a=b-2c\ [1-(b-c)^{-1}]$. This region leads to the stability requirements that $a \sim 2$, $c \approx 0$, and

$$a+3c-[(c+a)^2-8c]^{1/2}<2b<a+3c+[(c+a)^2-8c]^{1/2} \quad (1)$$

Several techniques may be applied to select numerical values for the controller gains (a, b, c) and the sample period (T). However, in each case the resulting gains and sample period must be selected so that they lie within the prescribed stability region. The first (case 1) to be considered is an application of the conventional pole placement technique. It is based on the premise that the dynamic behavior of the system is related closely to the location of the roots of its associated C.E.. The method shows both analytically and graphically the direct correlation between these roots and the control gains and sample period of the controller. The design technique then involves the specification of the C.E. root locations and the subsequent determination of the control system gains and sample period needed to attain these locations. The control system designer then must determine the system response resulting from using these numerical values and assess its adequacy. If it is not adequate, he usually relies on his experience to relocate the roots to improve the response in the manner desired for his particular system (i.e., faster settling time, lower peak overshoot, etc.). It is assumed that one wishes the pair of complex conjugate poles of the C.E. to dominate the dynamic response of the system. This response then will be modified by locating the third (real) root, where δ is its values in the z domain. Application of the referenced parameter plane technique permits one to map contours of constant damping ratio ζ as functions of the independent

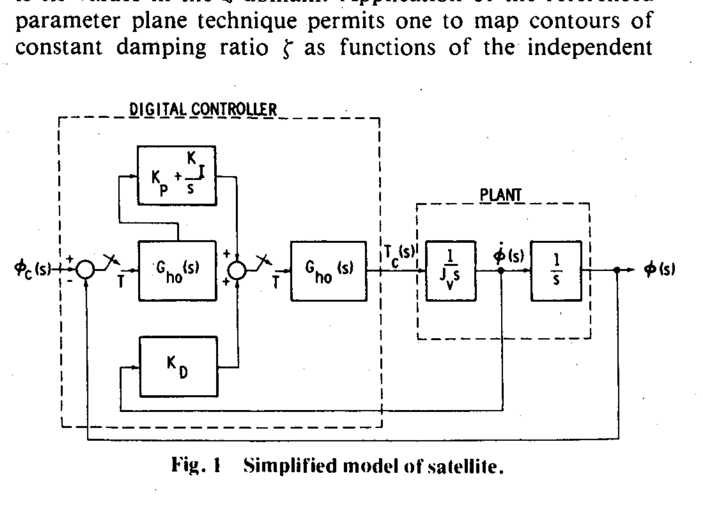

Fig. 1 Simplified model of satellite.

Table 1 Unit step responses

Case	T,sec	a	b	c	$\phi(T)$	$\phi(2T)$	$\phi(3T)$	$\phi(4T)$	$\phi(5T)$	$\phi(6T)$
1	0.2	1.41	0.673	0.1	0.673	1.49	1.51	1.26	1.07	0.99
2	1	1.95	1.5	0.091	1.5	1	1	1.18	0.92	1.12
3	1	1.78	0.6	0.015	0.6	1	1	~1	~1	~1
4	1	1.75	1.25	0.25	1.25	1.75	1	1	1	1

argument $\omega_n T$ (where ω_n is the natural frequency associated with the pair of complex conjugate roots) onto the a, b, c parameter space. If one wishes to map in two-dimensional parameter spaces, one may select iteratively values for c and plot ζ and δ contours on a b vs a parameter plane. In this Note, this is shown for $c=0.1$ (Fig. 2). In actual design practice, a number of values of c would be selected. Then the values of ω_n and ζ would be chosen. Computer design constraints (such as selecting as large a value of T as possible) then would be considered. Then an intersection of a δ-contour which gives a desired value for $\omega_n T$ would be found. In general, if $\delta \to 0$ its effect on the system dynamics is small compared to the effect of the pair of complex conjugate poles (placed by $\omega_n T$, ζ). Finally, one would check the response of the system using the values of a, b, c, and $\omega_n T$ (and hence K_p, K_i, K_d, and T) associated with the selected intersection. If it is unsatisfactory, reiterate the foregoing procedure, selecting another value of c

The system response at the sampled instants may be found from the closed loop transfer function in the z domain:

$$\phi(z)/\phi_c(z) = [bz^2 + 2cz + 2c - b]/\text{C.E.}(z)] \tag{2}$$

where

$$\text{C.E.}(z) = \sum_{j=0}^{3} \gamma_j z^j = 0$$

and where $\gamma_0 = a - b + 2c - 1$, $\gamma_1 = -2a + 2c + 3$, $\gamma_2 = a + b - 3$, and $\gamma_3 = 1$. Since the values of the roots of C.E. (z) have been selected tentatively in terms of ζ, $\omega_n T$, and δ, one may evaluate the time respone $\phi(T)$ to a given input $\phi_c(T)$ by any of the standard techniques (e.g., obtaining a partial fraction expansion and looking up the inverse z transforms, obtaining a power series in inverse powers of z, by evaluating the inverse of the complex convolution integral, etc.). For instance, the system response at sampled instants to a unit step input is

$$\phi(kT) = N(kT)\phi(T) - d^T\phi(kT), \quad k > 1 \tag{3}$$

where $\phi(kT) \equiv \{\phi[k-4)T], \phi[(k-3)T], \phi[(k-2)T], \phi[(k-1)T]\}^T$, $\phi(T) = b$, $d \equiv [\gamma_0, \gamma_0 - \gamma_1, \gamma_1 - \gamma_2, \gamma_2 - \gamma_3]^T$, $\phi(kT)$, is the system response (assumed to be zero for $t > 0$ at the kth sampled instand, and $N(kT) = 0$ except when $k = 2$ or 3: $N(2T) = 2c/b$ and $N(3T) = N(2T) - 1$.

As an example, assume that a spacecraft similar to the Space Telescope is to have a digital onboard controller.[5] Furthermore, assume that it is desired to have a controller natural frequency (ω_n) of 6 rad/sec, and that the moment of inertia (J_v) about the single axis considered in Ref. 5 (the Space Telescope V2 axis) is 4.295×10^4 kg-m^2. Assume that integral control is desired to drive to zero the effect of constant input disturbances. This means that a nonzero value of c is desired. If a value of ζ of $1/\sqrt{2}$ is selected, and if the value of δ is desired to be kept as close to zero as possible, it is found that, for $\delta > 0$, no intersections of the ζ-contour occur when $c > 0.2$. Hence, a value of $c = 0.1$ is chosen. The smallest value of δ whose corresponding δ line intersects the ζ-contour, with a reasonable factor of safety, is $\delta = 0.4$. The value of $\omega_n T = 1.2$ is on the ζ-contour at the intersection, yielding a

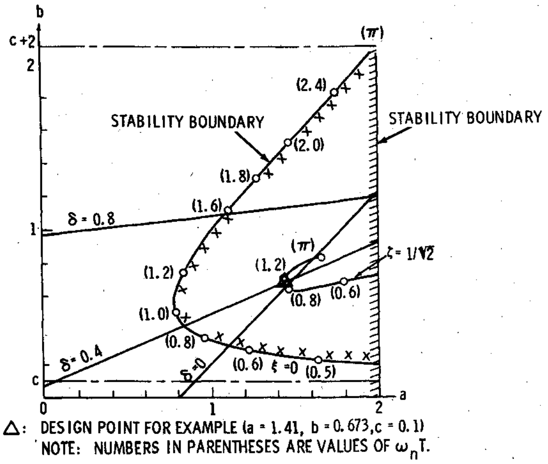

Fig. 2 Parameter plane plot portraying root locations ($c = 0.1$).

value of 0.20 sec for T. Corresponding values of a and b are 1.41 and 0.673, respectively. Knowledge of values for a, b, c, and T then permits one to calculate values for K_p, K_i, and K_d. If response studies show that a larger value of integral gain is deemed necessary or that the selected damping is unsatisfactory, a larger value of c (and different value of ζ) would be selected.

A second approach (case 2) is to determine explicitly the values of the response $\phi(T)$ at the first several sampled instants. Assume, for example, that the input command $\phi_c(T)$ is a unit step function and that the value of the response at the first sampled instant $\phi(T)$ is to be assigned. It may be shown that $\phi(T) = b$, so the desired value of $\phi(T)$ sets the value of b. Since two additional degrees of freedom exist in the system, one may assign values of $\phi(2T)$ and $\phi(3T)$, such as $\phi(2T) = \phi(3T) = 1$. If one now imposes the preceding conditions $[c(2T) = \phi(3T)]$ on the system, one readily may obtain the expressions

$$a = -(b^3 - 6b^2 + 13b - 4)/(b^2 - 4b + 1) \tag{4}$$

$$c = [1 + b(a + b - 4)]/2 \tag{5}$$

Now one sets an acceptable value of $\phi(T)$ which sets the value of b. Knowing b, one may find a and c from Eqs. (4) and (5). One only must insure that these values lie within the stability region (Fig. 2). If one selects a value for $\phi(T) = 1.5$, then $a = 1.955$, $b = 1.5$, $c = 0.091$. If one uses Shannon's sampling theorem as a constraint ($\omega T < \Pi$), one may prescribe a value for system frequency (ω). Using the foregoing Space Telescope example, $\omega \approx \omega_n/\sqrt{2} = 4.24$ rad/sec. Setting $\omega T = 3\Pi/4$ (to insure meeting Shannon's constraint) yields a value of $T \approx 1$ sec. If one wishes a lower value for $\phi(T) = 0.6$ (case 3), then $a = 1.7846$, $b = 0.6$, $c = 0.01539$.

Finally, one might wish to design a system whose step function response has zero steady-state error (case 4). Application of the sampled-data final-value theorem to Eq. (2) yields the requirements on the modified gains of $a = 7/4$, $b = 5/4$, and $c = ¼$. The system response at the sampling instants become $\phi(T) = 5/4$, $\phi(2T) = 7/4$, and $\phi(k/T) = 1$ for integer $k \geq 3$.

A comparison of the unit step responses is shown in Table 1. If one wishes to design a PID controller for a system with flexible appendages (such as Space Telescope solar panels), appendage dynamics would be characterized in modal coordinates, such as normal modes. Their inclusion increases the order of the characteristic equation, but the referenced parameter plane technique still could be applied to establish values for modified gains *a, b, c*. The resulting analytical complexity is being checked now by the author to determine if this complexity masks the usefulness of the technique described. If so, the technique still is useful for initial selection of gain values and sample period for preliminary design purposes.

Conclusions

A technique to aid the control system engineer in his selection of numerical values for satellite onboard digital computer gains and sample period has been presented. It is postulated that the technique may be extended to handle larger-order system to increase the fidelity of the system that is represented analytically.

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Yaw Induction by Mass Asymmetry

Charles H. Murphy*

U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Md.

Nomenclature

I_t = transverse moment of inertia of the shell
I_x = axial moment of inertia of the shell
K_j = absolute value of the jth modal arm, rad ($j = 1, 2, 3$)
l_ϵ = distance of unbalance along the axis of symmetry in front of the center of mass of the shell
m_ϵ = mass of unbalance
r_ϵ = distance of unbalance off the axis of symmetry
t = time
$\tilde\alpha$ = angle of attack in a nonrolling missile-fixed system
$\tilde\beta$ = angle of sideslip in a nonrolling missile-fixed system
λ_j = damping rate of the jth modal arm ($j = 1, 2$)
ϕ = roll angle of the shell (taken as zero at time zero)
ϕ_j = orientation angle of the jth modal arm ($j = 1, 2, 3$)
$(\dot{\ })$ = derivative of ( ) with respect to time
$(\ )_0$ = value of ( ) at time zero

Introduction

THE need for controlling launch yaws during development flight testing has been generally recognized by most aeroballisticians. A particularly dramatic example of this need is given by the 155-mm M483 shell. In January 1974, 20 of the M483's were fired at transonic launch Mach numbers and seven flew to less than 65% of full range.[1] During the remainder of 1974, a substantial engineering effort led to a modified version of the M483 which apparently did not have the transonic instability observed in January. Final proof tests of this improved version were planned for the following winter. During these tests, unmodified M483's were fired as controls, with the unexpected result that 48 successive unmodified M483's achieved full range, and the first short occurred for the 49th round![2] Since the shorts occur only when the launch maximum angle of attack exceeds 5°, the tube-projectile combination of 1975 apparently gave much smaller launch angles than the combination used in 1974. Since natural gun launch was, therefore, not a very reliable test, the validity of the M483 modification had to be established by artificially inducing yaws up to 16° and observing the resulting rapid damping of the angular motion.

Yaw usually is induced by means of modified muzzle brakes. Although this technique is reliable for moderate muzzle velocities, it can damage the shell or gun at high muzzle velocities. In this Note, we shall show how the introduction of a mass asymmetry can be a convenient yaw-induction technique.

Theory

As is shown in Refs. 3 and 4, the effect of a small dynamic unbalance is to add a third mode of oscillation to the usual two modes present in the shell's angular motion. This third mode causes a circular motion with frequency equal to the shell's spin and a magnitude equal to the angle between the unbalanced shell's normal axis of inertia and the balanced shell's normal axis of inertia. For the unbalance introduced by a small mass m_ϵ located a distance r_ϵ off the axis of symmetry and a distance l_ϵ along the axis of symmetry in front of the shell's center of mass, this angle is

$$K_3 = m_\epsilon l_\epsilon r_\epsilon / (I_t - I_x) \tag{1}$$

The complete tricyclic equation for pitching and yawing motion is

$$\tilde\beta + i\tilde\alpha = K_1 \exp(i\phi_1) + K_2 \exp(i\phi_2) + K_3 \exp[\,(i(\phi + \phi_{30})] \tag{2}$$

where

$$K_j = K_{j0} \exp(\lambda_j t) \qquad (j = 1,2)$$

$$\phi_j = \dot\phi_j t + \phi_{j0} \qquad (\dot\phi_1 > \dot\phi_2,\ j = 1,2)$$

With the reasonable assumption of small initial angle and the approximation of small angular velocity, Ref. 4 shows that

$$K_1 = [\,(\dot\phi - \dot\phi_2) / (\dot\phi_1 - \dot\phi_2)\,] K_3 \tag{3}$$

$$K_2 = [\,(\dot\phi - \dot\phi_1) / (\dot\phi_1 - \dot\phi_2)\,] K_3 \tag{4}$$

For gyroscopically stable shell, the frequency factors in Eqs. (3) and (4) usually exceed eight, and thus an unbalance angle of 0.5° can cause at least 4° in the fast and slow motion

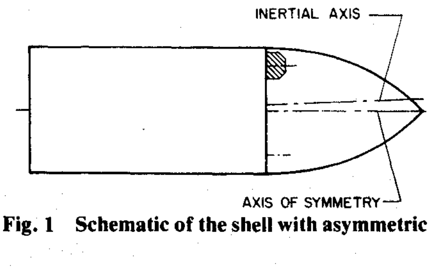

Fig. 1 Schematic of the shell with asymmetric mass.

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*Chief, Launch and Flight Division. Associate Fellow AIAA.